Wahlaufgaben

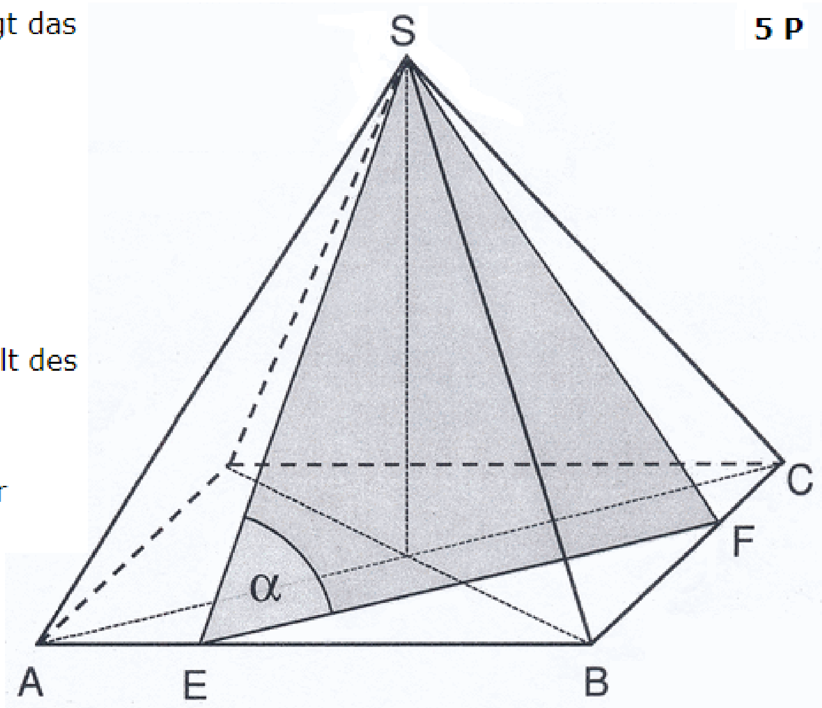
Aufgabe 2021 B/2b:

5 P

In einer quadratischen Pyramide liegt das gleichschenklige Dreieck EFS.
Es gilt:

$$\overline{AB} = \overline{EF} = 12,6\text{cm}$$
$$\alpha = 72,0^\circ$$
$$\overline{EF} \parallel \overline{AC}$$

- Berechnen Sie den Flächeninhalt des Dreiecks EFS.
- Berechnen Sie das Volumen der quadratischen Pyramide.



Strategie 2021 B/2b:

Gegeben:

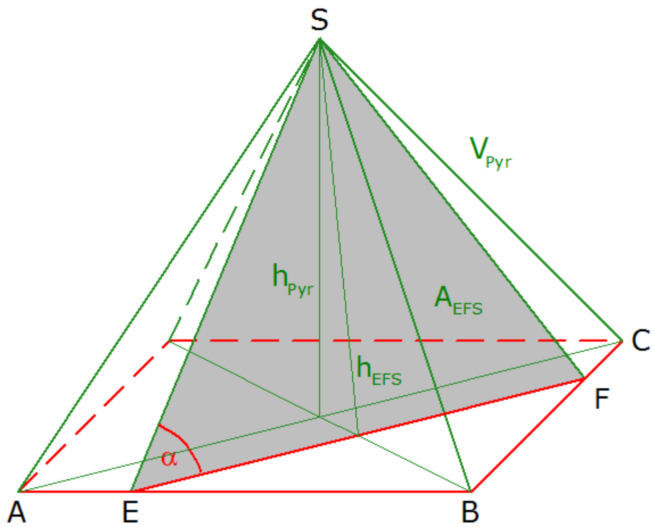
$$\overline{AB} = \overline{EF} = 12,6\,\text{cm}$$
$$\alpha = 72,0^\circ$$
$$\overline{EF} \parallel \overline{AC}$$

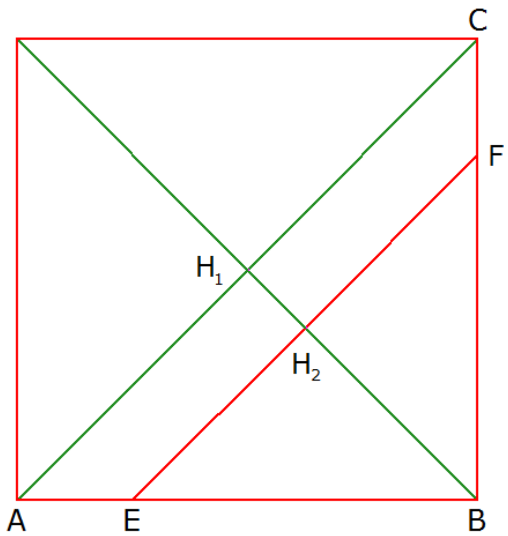
Gesucht:

A_{EFS}

V_{Pyr}

Skizze:





Strategie 2021 B/2b:

Struktogramm:

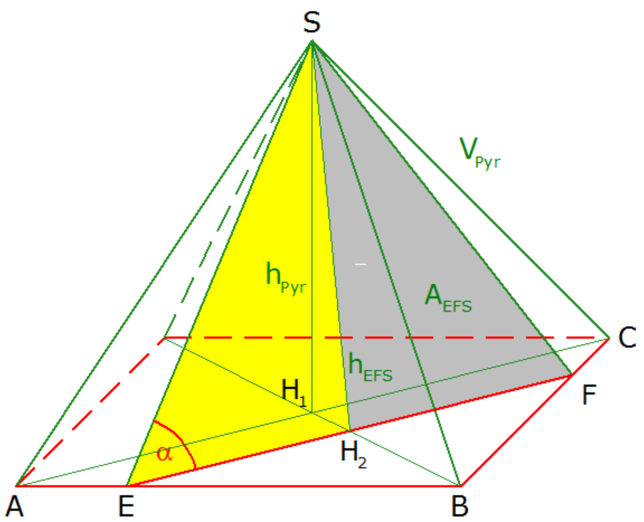
2 A_{EFS}: $A_{EFS} = \frac{1}{2} \cdot \overline{EF} \cdot h_{EFS}$	**9** V_{Pyr}: $V_{Pyr} = \frac{1}{3} \cdot \overline{AB}^2 \cdot h_{Pyr}$
1	2
1 h_{EFS}: $\tan\alpha = \dfrac{h_{EFS}}{\frac{1}{2} \cdot \overline{EF}}$	**8** h_{Pyr}: $h_{Pyr}^2 + \overline{H_1H_2}^2 = h_{EFS}^2$

3 4 7

7 $\overline{H_1H_2}$: $\overline{H_1H_2} = \overline{BH_1} - \overline{BH_2}$	**6** $\overline{BH_2}$: $\dfrac{\overline{EH_2}}{\overline{AH_1}} = \dfrac{\overline{BH_2}}{\overline{BH_1}}$	**3** $\overline{BH_1}$: $\left(2 \cdot \overline{BH_1}\right)^2 = \overline{AB}^2 + \overline{AB}^2$
5	6	
5 $\overline{EH_2}$: $\overline{EH_2} = \frac{1}{2} \cdot \overline{EF}$	**4** $\overline{AH_1}$: $\overline{AH_1} = \overline{BH_1}$	

Lösung 2021 B/2b:

1. Berechnung der Dreieckshöhe h_{EFS}:



$\tan\alpha = \dfrac{\text{Gegenkathete}}{\text{Ankathete}} = \dfrac{h_{EFS}}{\overline{EH_2}}$ Tangensfunktion im rechtwinkligen gelben Teildreieck

$\tan 72° = \dfrac{h_{EFS}}{\frac{12,6}{2}}$

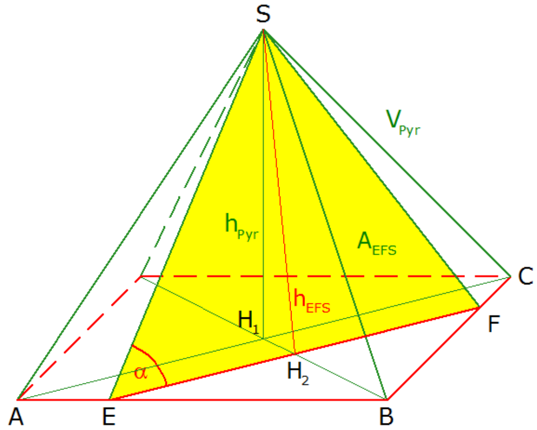
$3,0777 = \dfrac{h_{EFS}}{6,3}$ Seiten tauschen

$\dfrac{h_{EFS}}{6,3} = 3,0777 \quad | \cdot 6,3$

$\underline{h_{EFS} = 19,39\,\text{cm}}$

Lösung 2021 B/2b:

2. Berechnung der Dreiecksfläche A_{EFS}:

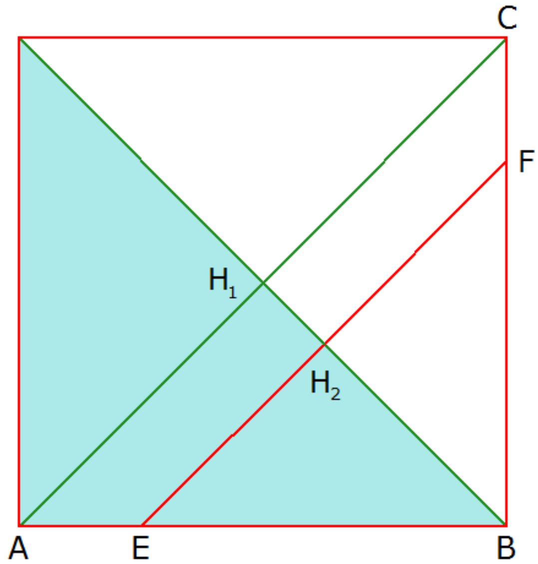


$$A_{EFS} = \frac{1}{2} \cdot \overline{EF} \cdot h_{EFS}$$ Formel Dreiecksfläche

$$A_{EFS} = \frac{1}{2} \cdot 12{,}6 \cdot 19{,}39$$

$$\underline{\underline{A_{EFS} = 122{,}16\,cm^2}}$$

3. Berechnung der Strecke $\overline{BH_1}$:



$$\left(2 \cdot \overline{BH_1}\right)^2 = \overline{AB}^2 + \overline{AB}^2$$ Pythagoras im hellblauen rechtwinkligen Teildreieck

$$4 \cdot \overline{BH_1}^2 = 12{,}6^2 + 12{,}6^2$$

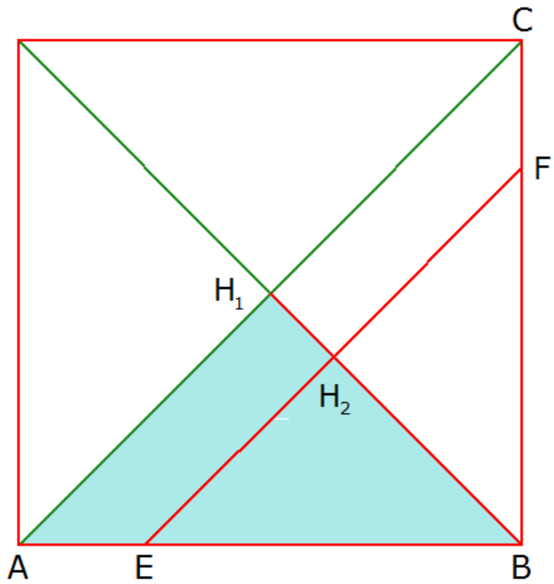
$$4 \cdot \overline{BH_1}^2 = 158{,}76 + 158{,}76$$

$$4 \cdot \overline{BH_1}^2 = 317{,}52 \qquad |:4$$

$$\overline{BH_1}^2 = 79{,}38 \qquad |\sqrt{}$$

$$\underline{\overline{BH_1} = 8{,}91\,cm}$$

4. Berechnung der Strecke $\overline{AH_1}$:



$$\overline{AH_1} = \overline{BH_1}$$

$$\underline{\overline{AH_1} = 8{,}91\,cm}$$

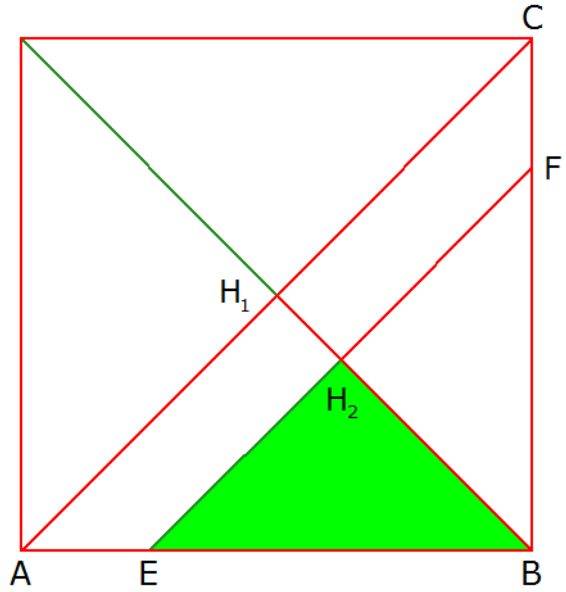
Lösung 2021 B/2b:

5. Berechnung der Strecke $\overline{EH_2}$:

$$\overline{EH_2} = \frac{1}{2} \cdot \overline{EF}$$

$$\overline{EH_2} = \frac{1}{2} \cdot 12,6$$

$$\overline{EH_2} = 6,3\,\text{cm}$$



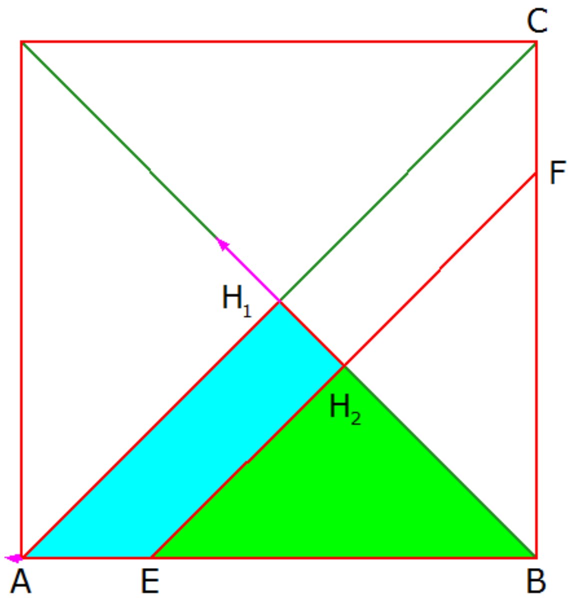
6. Berechnung der Strecke $\overline{BH_2}$:

2. Strahlensatz mit Z = B

$$\frac{\overline{BH_2}}{\overline{BH_1}} = \frac{\overline{EH_2}}{\overline{AH_1}}$$

$$\frac{\overline{BH_2}}{8,91} = \frac{6,3}{8,91} \quad \Big| \cdot 8,91$$

$$\overline{BH_2} = 6,3\,\text{cm}$$

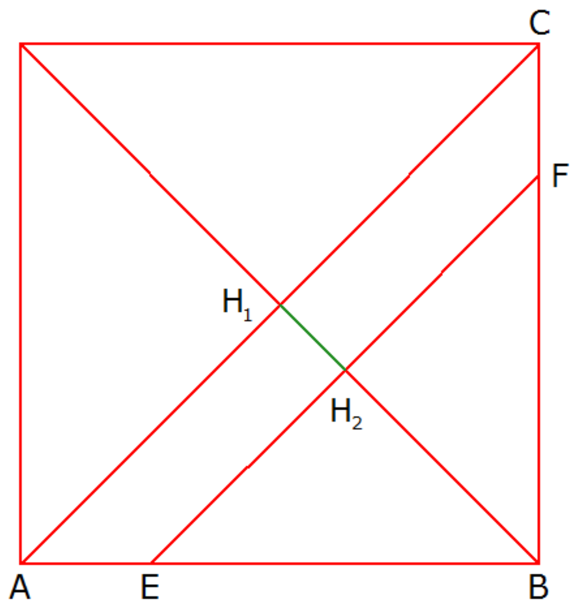


7. Berechnung der Strecke $\overline{H_1H_2}$:

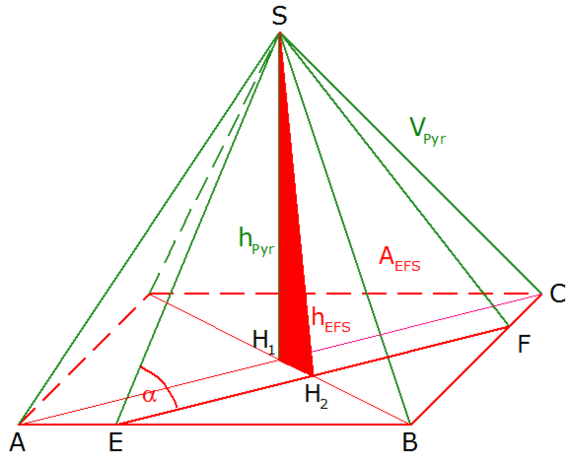
$$\overline{H_1H_2} = \overline{BH_1} - \overline{BH_2}$$

$$\overline{H_1H_2} = 8,91 - 6,3$$

$$\overline{H_1H_2} = 2,61\,\text{cm}$$



Lösung 2021 B/2b:



8. Berechnung der Pyramidenhöhe h_{Pyr}:

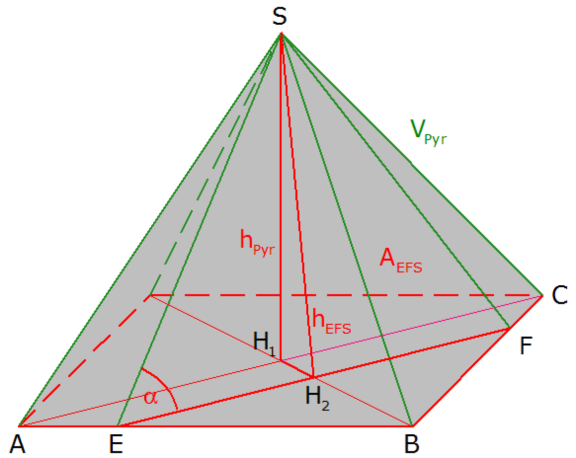
$h_{Pyr}^2 + \overline{H_1H_2}^2 = h_{EFS}^2$ — Pythagoras im rechtwinkligen roten Teildreieck

$h_{Pyr}^2 + 2,61^2 = 19,39^2$

$h_{Pyr}^2 + 6,8121 = 375,9721 \quad \mid -6,8121$

$h_{Pyr}^2 = 369,16 \quad \mid \sqrt{\ }$

$\underline{h_{Pyr} = 19,21\,cm}$



9. Berechnung des Pyramidenvolumens V_{Pyr}:

$V_{Pyr} = \frac{1}{3} \cdot \overline{AB}^2 \cdot h_{Pyr}$ — Formel Pyramidenvolumen

$V_{Pyr} = \frac{1}{3} \cdot 12,6^2 \cdot 19,21$

$\underline{\underline{V_{Pyr} = 1016,6\,cm^3}}$