Wahlaufgaben

Aufgabe 2016 W1a:

Die Eckpunkte des Vierecks ABCD liegen auf den Parallelen g und h.
Die Parallelen haben einen Abstand von 9,0 cm.

5,5 P

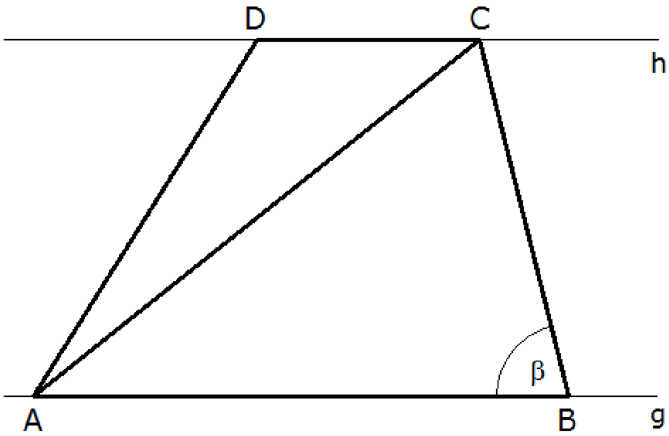
Es gilt:

$\overline{AD} = 10{,}4\,\text{cm}$

$\beta = 70{,}0^\circ$

$\overline{AB} = \overline{AC}$

Berechnen Sie den Umfang des Vierecks ABCD.



Strategie 2016 W1a:

Gegeben:

Trapez ABCD

$\overline{AD} = 10{,}4\,\text{cm}$

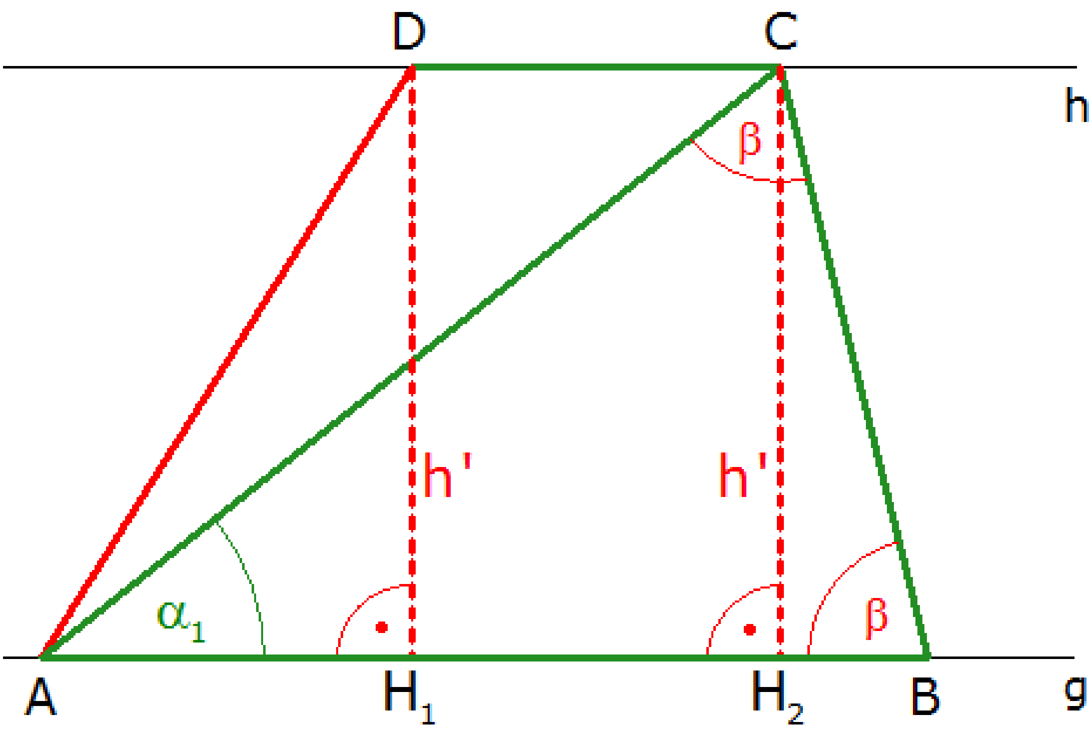
$\beta = 70^\circ$

$\overline{AB} = \overline{AC}$

Gesucht:

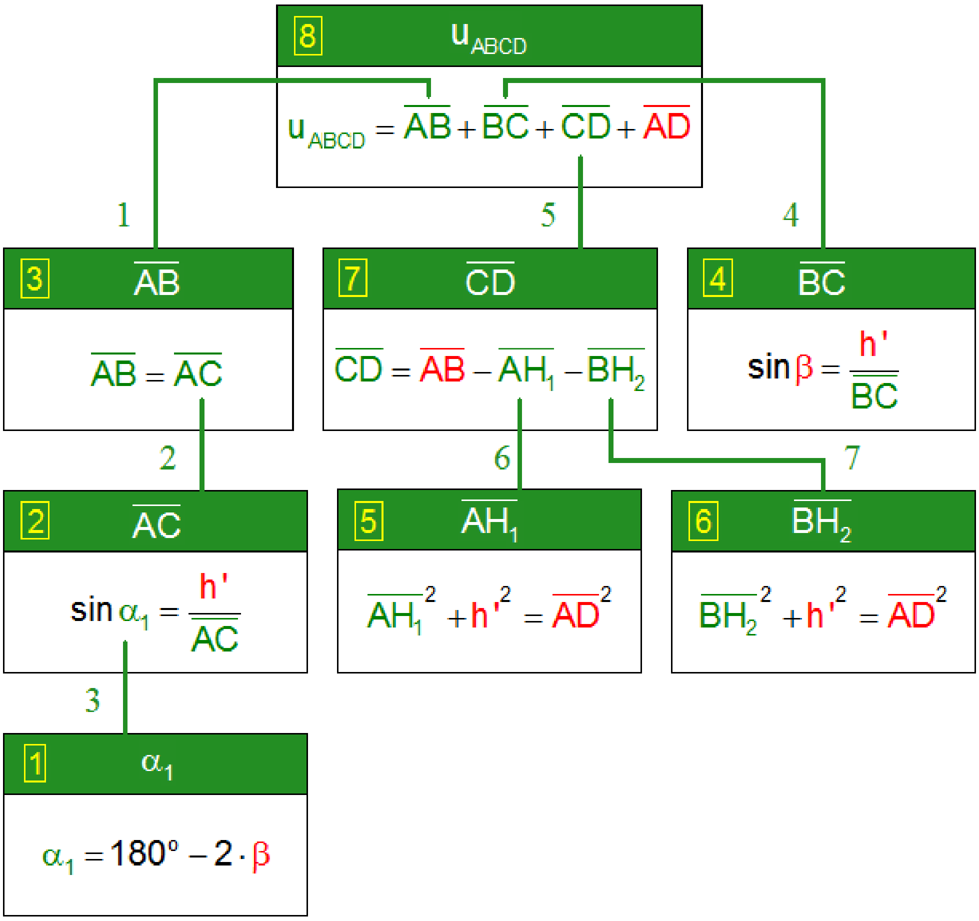
u_{ABCD}

Skizze:



Strategie 2016 W1a:

Struktogramm:



Lösung 2016 W1a:

1. Berechnung des Winkels α_1:

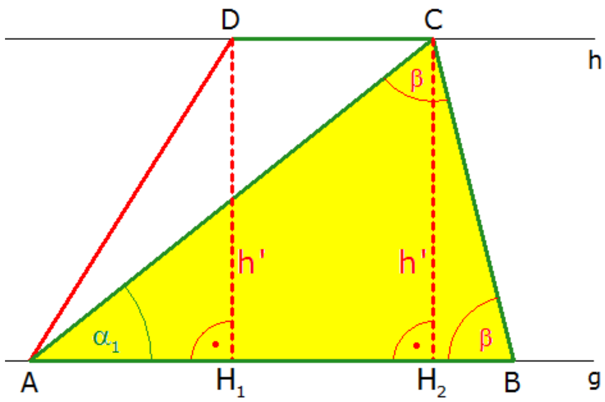
$$\alpha_1 = 180° - 2 \cdot \beta$$

Das gelbe Teildreieck ABC ist gleichschenklig; Basiswinkel sind gleich groß

$$\alpha_1 = 180° - 2 \cdot 70°$$

$$\alpha_1 = 180° - 140°$$

$$\underline{\alpha_1 = 40°}$$



2. Berechnung der Strecke $\overline{AC}$:

$$\sin\alpha_1 = \frac{\text{Gegenkathete}}{\text{Hypotenuse}} = \frac{h'}{\overline{AC}}$$

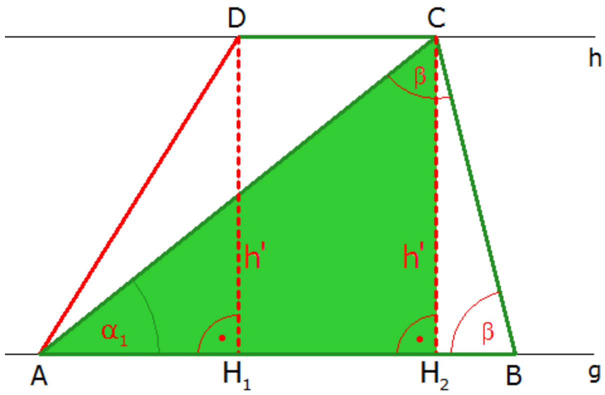
Sinusfunktion im rechtwinkligen grünen Teildreieck ACH_2

$$\sin 40° = \frac{9}{\overline{AC}}$$

$$0,6428 = \frac{9}{\overline{AC}} \qquad | \cdot \overline{AC}$$

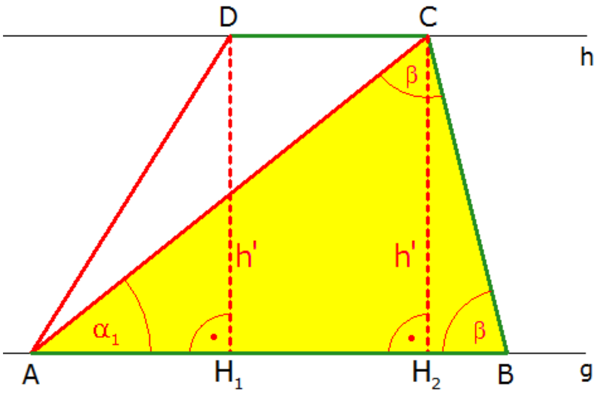
$$\overline{AC} \cdot 0,6428 = 9 \qquad | : 0,6428$$

$$\underline{\overline{AC} = 14\,\text{cm}}$$



Lösung 2016 W1a:

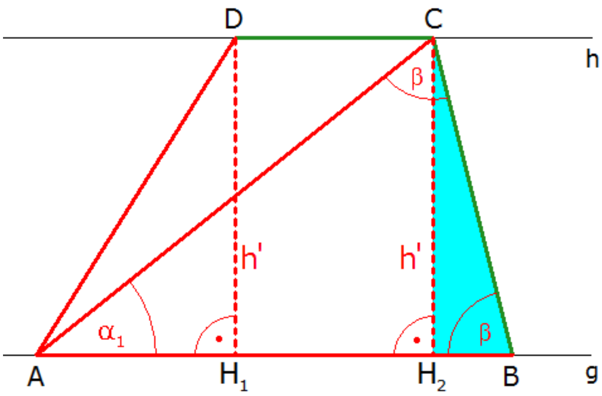
3. Berechnung der Strecke $\overline{AB}$:



$$\overline{AB} = \overline{AC}$$

$$\underline{\overline{AB} = 14\,cm}$$

4. Berechnung der Strecke $\overline{BC}$:



$$\sin\beta = \frac{\text{Gegenkathete}}{\text{Hypotenuse}} = \frac{h'}{\overline{BC}}$$

Sinusfunktion im rechtwinkligen hellblauen Teildreieck BCH_2

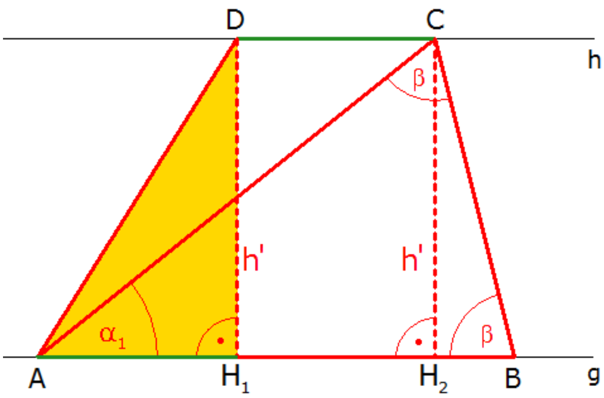
$$\sin 70^\circ = \frac{9}{\overline{BC}}$$

$$0,9397 = \frac{9}{\overline{BC}} \quad \Big|\cdot \overline{BC}$$

$$\overline{BC} \cdot 0,9397 = 9 \quad \Big|:0,9397$$

$$\underline{\overline{BC} = 9,58\,cm}$$

5. Berechnung der Strecke $\overline{AH_1}$:



$$\overline{AH_1}^2 + h'^2 = \overline{AD}^2$$

Pythagoras im rechtwinkligen goldfarbenen Teildreieck ADH_1

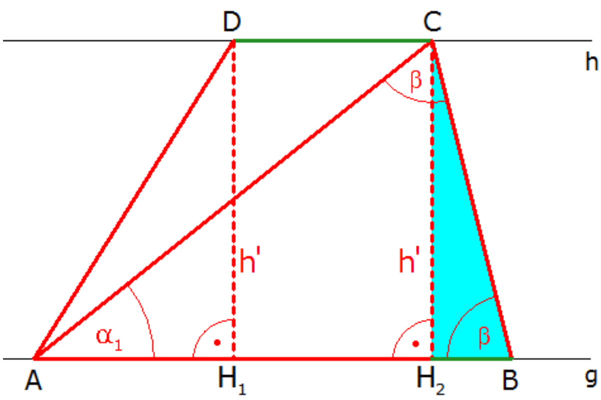
$$\overline{AH_1}^2 + 9^2 = 10,4^2$$

$$\overline{AH_1}^2 + 81 = 108,16 \quad \Big| -81$$

$$\overline{AH_1}^2 = 27,16 \quad \Big| \sqrt{\ }$$

$$\underline{\overline{AH_1} = 5,21\,cm}$$

6. Berechnung der Strecke $\overline{BH_2}$:



$$\overline{BH_2}^2 + h'^2 = \overline{BC}^2$$

Pythagoras im rechtwinkligen hellblauen Teildreieck BCH_2

$$\overline{BH_2}^2 + 9^2 = 9,58^2$$

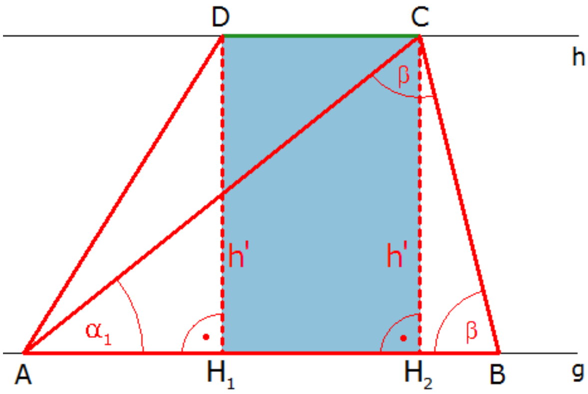
$$\overline{BH_2}^2 + 81 = 91,7764 \quad \Big| -81$$

$$\overline{BH_2}^2 = 10,7764 \quad \Big| \sqrt{\ }$$

$$\underline{\overline{BH_2} = 3,28\,cm}$$

Lösung 2016 W1a:

7. Berechnung der Strecke $\overline{CD}$:

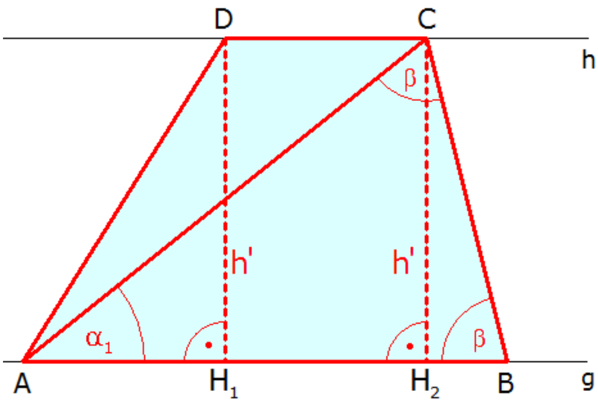


$$\overline{CD} = \overline{AB} - \overline{AH_1} - \overline{BH_2}$$

$$\overline{CD} = 14 - 5,21 - 3,28$$

$$\underline{\overline{CD} = 5,51\,\text{cm}}$$

8. Berechnung des Viereckumfangs u_{ABCD}:



$$u_{ABCD} = \overline{AB} + \overline{BC} + \overline{CD} + \overline{AD}$$

$$u_{ABCD} = 14 + 9,58 + 5,51 + 10,4$$

$$\underline{\underline{u_{ABCD} = 39,49\,\text{cm}}}$$