Wahlaufgaben

Aufgabe 2013 W1b:

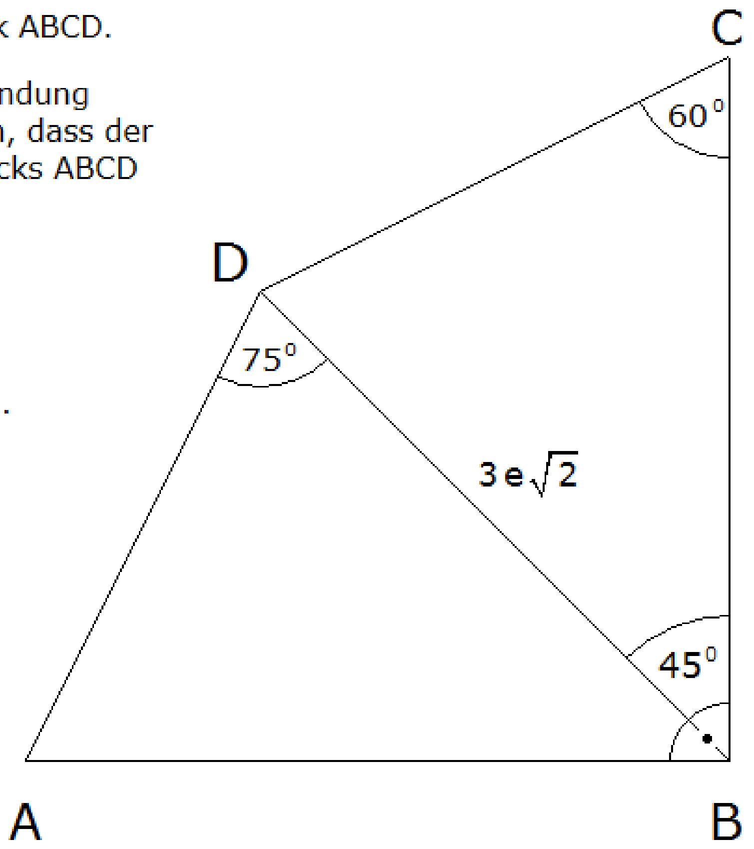
4 P

Gegeben ist das Viereck ABCD.

Weisen Sie ohne Verwendung gerunderter Werte nach, dass der Flächeninhalt des Vierecks ABCD mit der Formel

$$A = 3e^2\left(3+\sqrt{3}\right)$$

berechnet werden kann.



Strategie 2013 W1b:

<u>Gegeben:</u>

Viereck ABCD

$\beta = 90^0$

$\beta_2 = 45^0$

$\delta = 75^0$

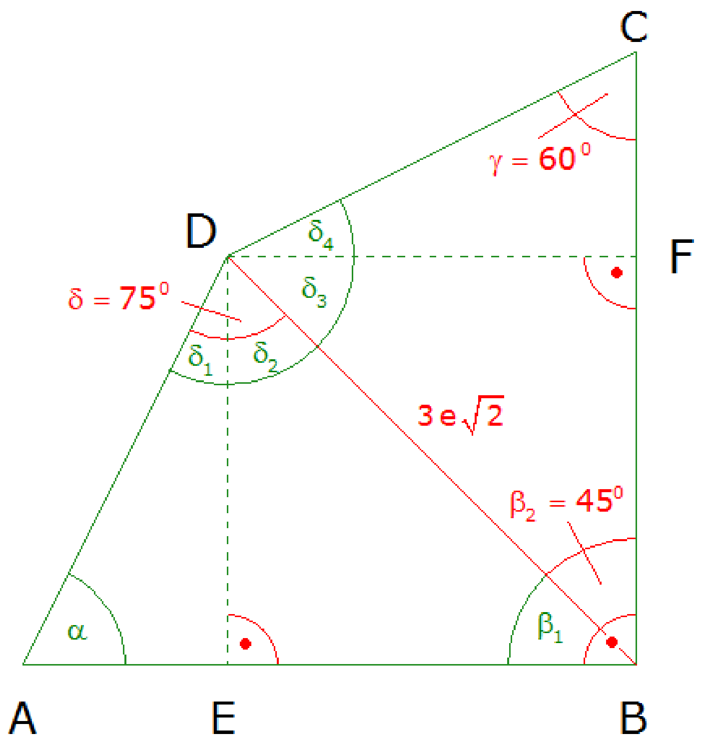
$\gamma = 60^0$

$\overline{BD} = 3e\sqrt{2}$

<u>Gesucht:</u>

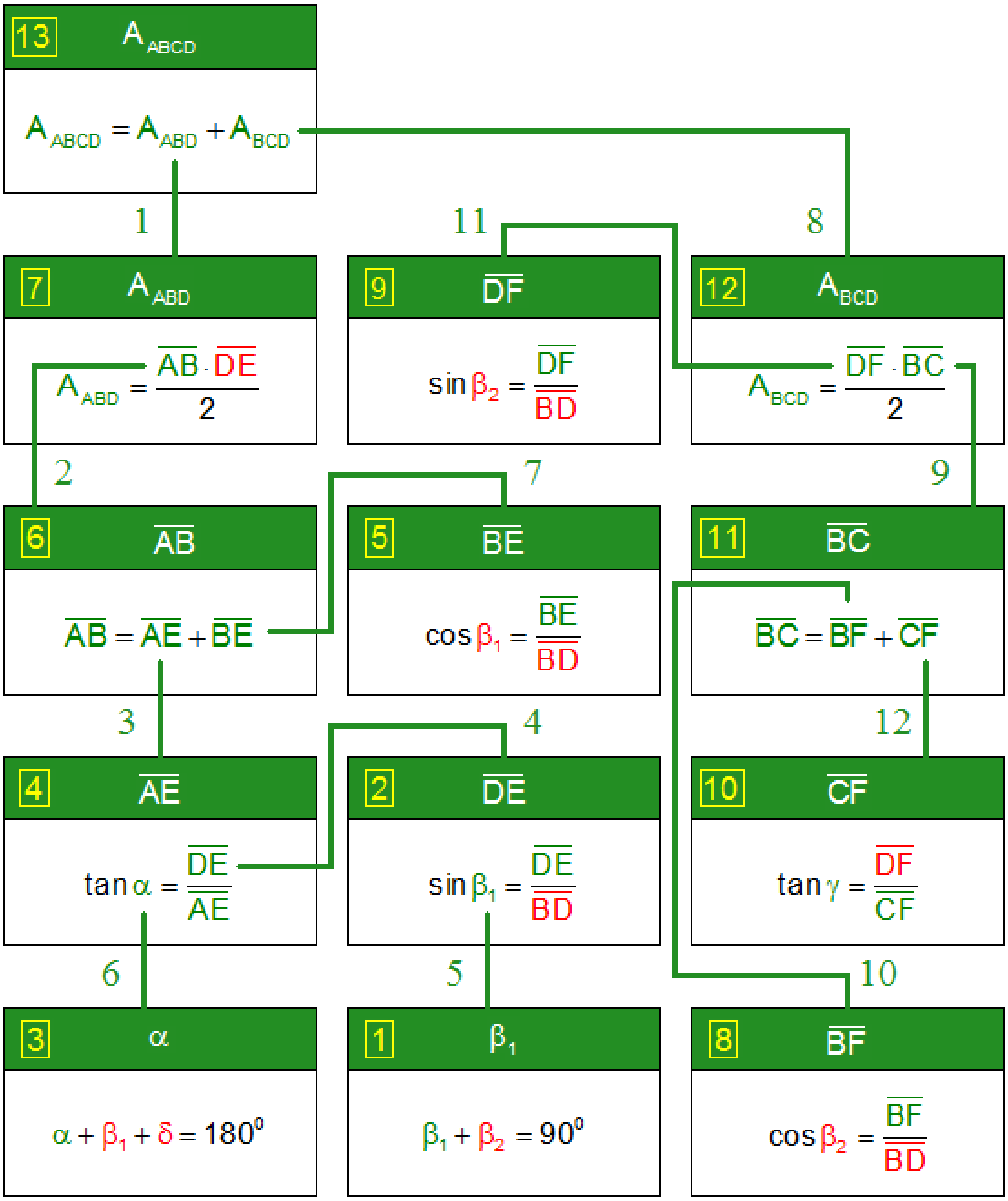
$A_{ABCD} = 3e^2\left(3+\sqrt{3}\right)$

<u>Skizze:</u>

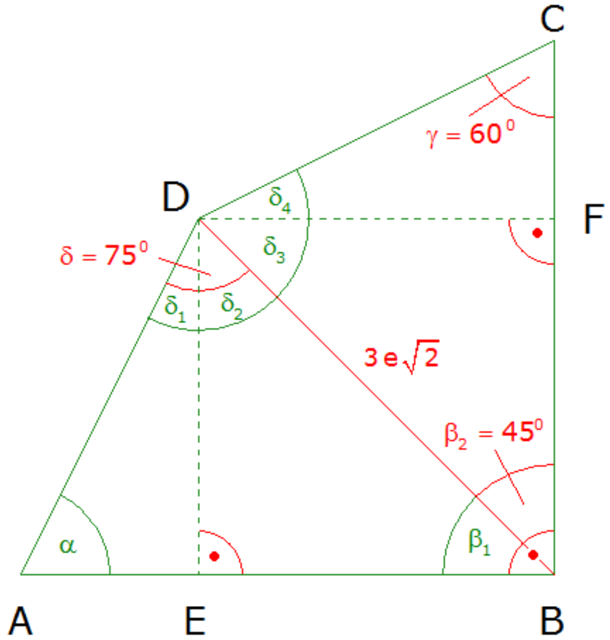


Strategie 2013 W1b

Struktogramm:



Lösung 2013 W1b:



1. Berechnung des Winkels β_1:

$$\beta_1 + \beta_2 = 90^0$$

$$\beta_1 + 45^0 = 90^0 \quad \Big| -45^0$$

$$\underline{\beta_1 = 45^0}$$

2. Berechnung der Strecke $\overline{DE}$:

$$\sin\beta_1 = \frac{\text{Gegenkathete}}{\text{Hypotenuse}} = \frac{\overline{DE}}{\overline{BD}}$$

Sinusfunktion im rechtwinkligen gelben Teildreieck BDE

$$\sin 45^0 = \frac{\overline{DE}}{3e\sqrt{2}}$$

$$\sin 45^0 = \frac{1}{2}\sqrt{2}$$

$$\frac{1}{2}\sqrt{2} = \frac{\overline{DE}}{3e\sqrt{2}}$$

Seiten tauschen

$$\frac{\overline{DE}}{3e\sqrt{2}} = \frac{1}{2}\sqrt{2} \quad \Big| \cdot 3e\sqrt{2}$$

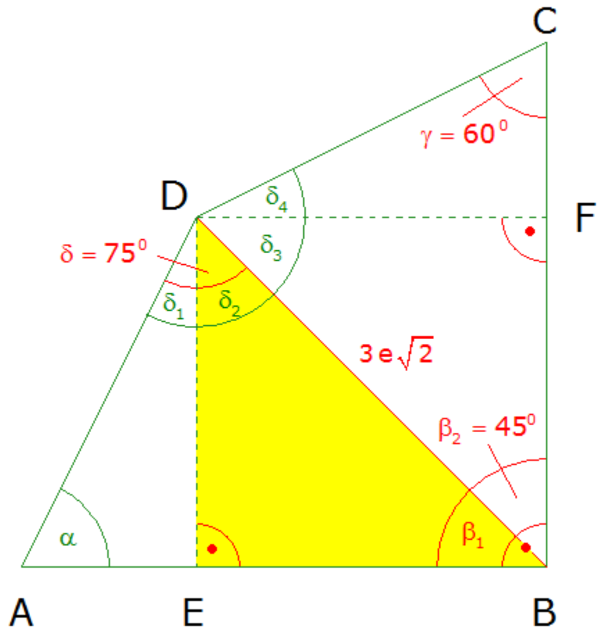
$$\overline{DE} = \frac{1}{2}\sqrt{2}\cdot 3e\sqrt{2}$$

$$\overline{DE} = \frac{1}{2}\sqrt{2}\cdot\sqrt{2}\cdot 3e$$

$$\overline{DE} = \frac{1}{2}\cdot 2\cdot 3e$$

$$\overline{DE} = \frac{1}{2}\cdot 2\cdot 3e$$

$$\underline{\overline{DE} = 3e}$$



Lösung 2013 W1b:

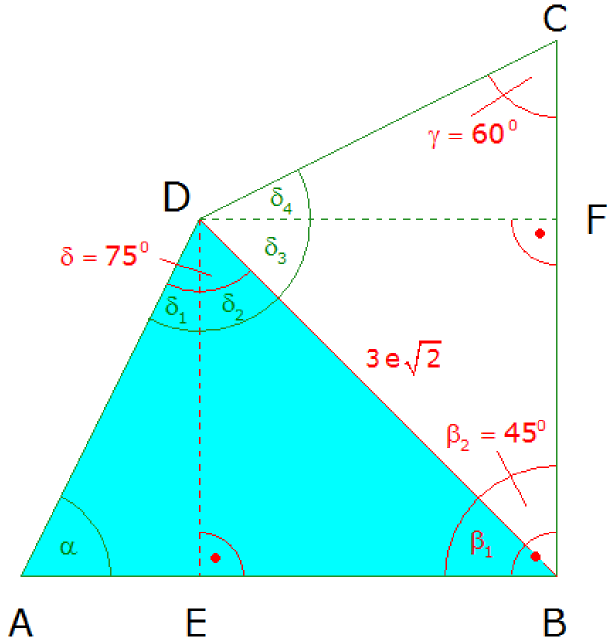
3. Berechnung des Winkels α:

$\alpha + \beta_1 + \delta = 180^0$ Winkelsumme im hellblauen Teildreieck ABD

$\alpha + 45^0 + 75^0 = 180^0$

$\alpha + 120^0 = 180^0 \quad | -120^0$

$\underline{\alpha = 60^0}$



4. Berechnung der Strecke $\overline{AE}$:

$\tan\alpha = \frac{\text{Gegenkathete}}{\text{Ankathete}} = \frac{\overline{DE}}{\overline{AE}}$ Tangensfunktion im rechtwinkligen grünen Teildreieck AED

$\tan 60^0 = \frac{3e}{\overline{AE}}$ $\quad \tan 60^0 = \sqrt{3}$

$\sqrt{3} = \frac{3e}{\overline{AE}} \quad | \cdot \overline{AE}$

$\sqrt{3} \cdot \overline{AE} = 3e \quad | : \sqrt{3}$

$\overline{AE} = \frac{3e}{\sqrt{3}}$

$\overline{AE} = \frac{3e \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}}$ Nenner rational machen

$\overline{AE} = \frac{3e \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}}$

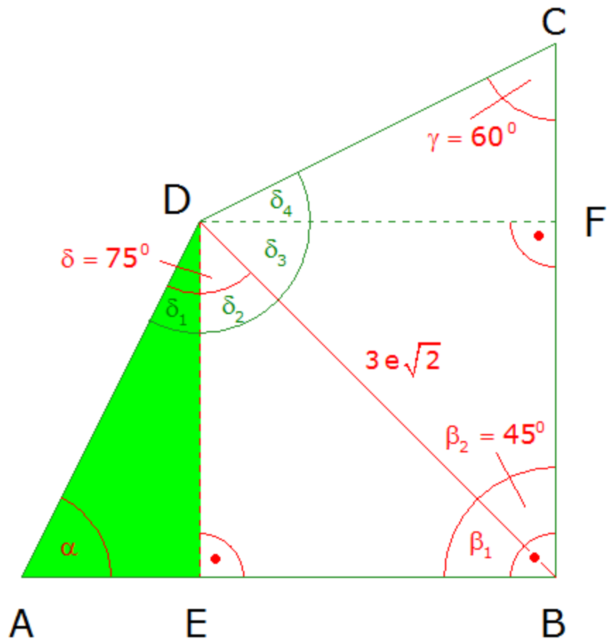
$\overline{AE} = \frac{3e \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}}$

$\overline{AE} = \frac{3e \cdot \sqrt{3}}{3}$

$\overline{AE} = \frac{3e \cdot \sqrt{3}}{3}$

$\overline{AE} = \frac{3 \cdot e \cdot \sqrt{3}}{3}$ Bruch kürzen

$\underline{\overline{AE} = e\sqrt{3}}$



Lösung 2013 W1b:

5. Berechnung der Strecke $\overline{BE}$:

$$\cos\beta_1 = \frac{\text{Ankathete}}{\text{Hypotenuse}} = \frac{\overline{BE}}{\overline{BD}}$$

Kosinusfunktion im rechtwinkligen gelben Teildreieck BDE

$$\cos 45^0 = \frac{\overline{BE}}{3e\sqrt{2}}$$

$\cos 45^0 = \frac{1}{2}\sqrt{2}$

$$\frac{1}{2}\sqrt{2} = \frac{\overline{BE}}{3e\sqrt{2}}$$

Seiten tauschen

$$\frac{\overline{BE}}{3e\sqrt{2}} = \frac{1}{2}\sqrt{2} \quad \Big| \cdot 3e\sqrt{2}$$

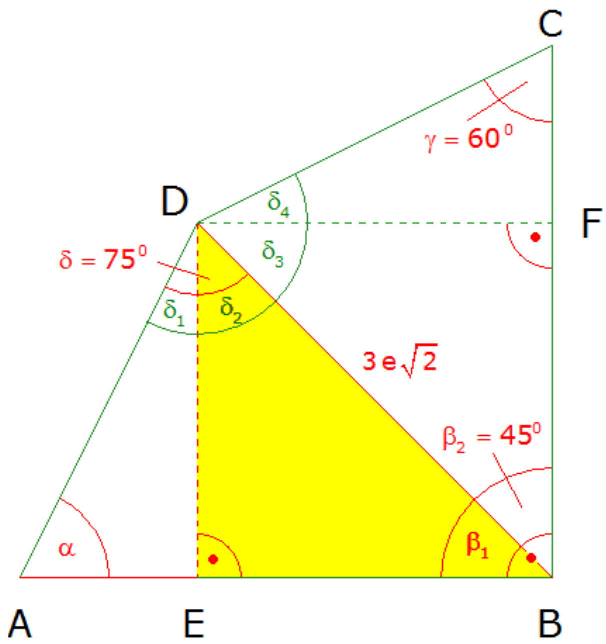
$$\overline{BE} = \frac{1}{2}\sqrt{2}\cdot 3e\sqrt{2}$$

$$\overline{BE} = \frac{1}{2}\sqrt{2}\cdot\sqrt{2}\cdot 3e$$

$$\overline{BE} = \frac{1}{2}\cdot 2\cdot 3e$$

$$\overline{BE} = \frac{1}{2}\cdot 2\cdot 3e$$

$$\underline{\overline{BE} = 3e}$$



6. Berechnung der Strecke $\overline{AB}$:

$$\overline{AB} = \overline{AE} + \overline{BE}$$

siehe hellblaues Teildreieck ABD

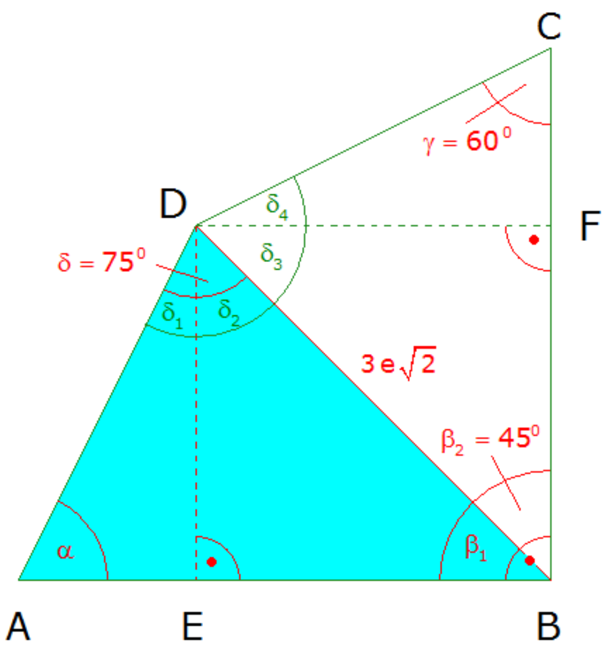
$$\overline{AB} = e\sqrt{3} + 3e$$

$$\overline{AB} = e\cdot\sqrt{3} + 3\cdot e$$

gemeinsamen Faktor ausklammern

$$\overline{AB} = e\left(\sqrt{3}+3\right)$$

$$\underline{\overline{AB} = e\left(\sqrt{3}+3\right)}$$



Lösung 2013 W1b:

7. Berechnung der Dreiecksfläche A_{ABD}:

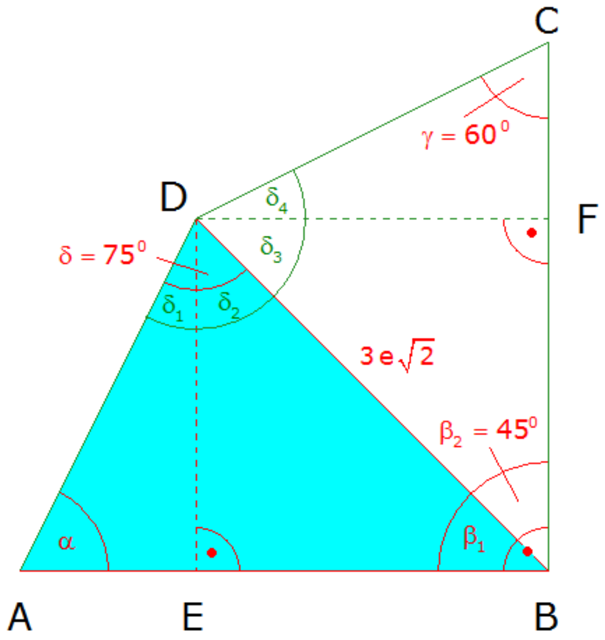
$$A_{ABD} = \frac{g \cdot h}{2} = \frac{\overline{AB} \cdot \overline{DE}}{2}$$

Flächenformel allgemeines Dreieck

$$A_{ABD} = \frac{e\left(\sqrt{3}+3\right) \cdot 3e}{2}$$

$$A_{ABD} = \frac{3e^2\left(\sqrt{3}+3\right)}{2}$$

$$\underline{A_{ABD} = 1{,}5e^2\left(\sqrt{3}+3\right)}$$



8. Berechnung der Strecke $\overline{BF}$:

$$\cos\beta_2 = \frac{\text{Ankathete}}{\text{Hypotenuse}} = \frac{\overline{BF}}{\overline{BD}}$$

Kosinusfunktion im rechtwinkligen hellgrauen Teildreieck BFD

$$\cos 45^0 = \frac{\overline{BF}}{3e\sqrt{2}}$$

$\cos 45^0 = \frac{1}{2}\sqrt{2}$

$$\frac{1}{2}\sqrt{2} = \frac{\overline{BF}}{3e\sqrt{2}}$$

Seiten tauschen

$$\frac{\overline{BF}}{3e\sqrt{2}} = \frac{1}{2}\sqrt{2} \qquad \left| \cdot 3e\sqrt{2}\right.$$

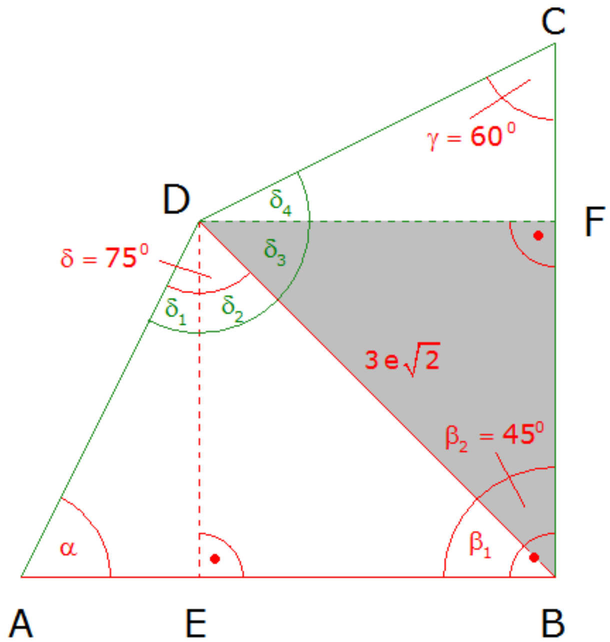
$$\overline{BF} = \frac{1}{2}\sqrt{2} \cdot 3e\sqrt{2}$$

$$\overline{BF} = \frac{1}{2}\sqrt{2} \cdot \sqrt{2} \cdot 3e$$

$$\overline{BF} = \frac{1}{2} \cdot 2 \cdot 3e$$

$$\overline{BF} = \frac{1}{2} \cdot 2 \cdot 3e$$

$$\underline{\overline{BF} = 3e}$$



Lösung 2013 W1b:

9. Berechnung der Strecke $\overline{DF}$:

$$\sin\beta_2 = \frac{\text{Gegenkathete}}{\text{Hypotenuse}} = \frac{\overline{DF}}{\overline{BD}}$$

Sinusfunktion im rechtwinkligen hellgrauen Teildreieck BFD

$$\sin 45^0 = \frac{\overline{DF}}{3e\sqrt{2}}$$

$$\sin 45^0 = \frac{1}{2}\sqrt{2}$$

$$\frac{1}{2}\sqrt{2} = \frac{\overline{DF}}{3e\sqrt{2}}$$

Seiten tauschen

$$\frac{\overline{DF}}{3e\sqrt{2}} = \frac{1}{2}\sqrt{2} \qquad \left| \cdot 3e\sqrt{2}\right.$$

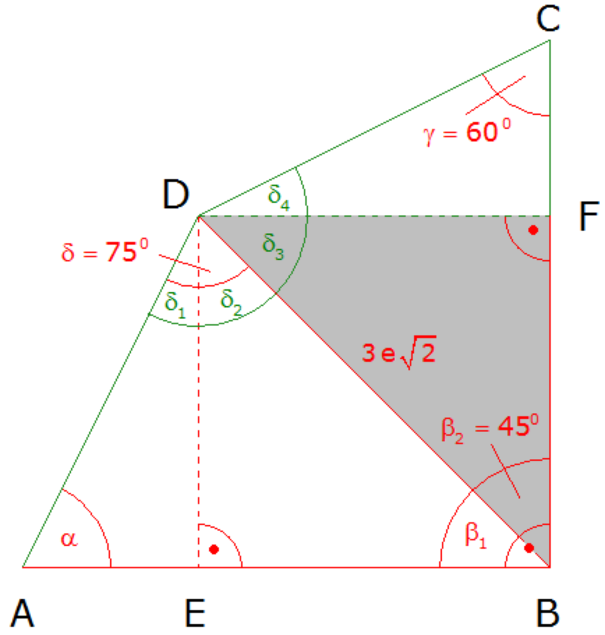
$$\overline{DF} = \frac{1}{2}\sqrt{2}\cdot 3e\sqrt{2}$$

$$\overline{DF} = \frac{1}{2}\sqrt{2}\cdot\sqrt{2}\cdot 3e$$

$$\overline{DF} = \frac{1}{2}\cdot 2\cdot 3e$$

$$\overline{DF} = \frac{1}{2}\cdot 2\cdot 3e$$

$$\underline{\overline{DF} = 3e}$$



10. Berechnung der Strecke $\overline{CF}$:

$$\tan\gamma = \frac{\text{Gegenkathete}}{\text{Ankathete}} = \frac{\overline{DF}}{\overline{CF}}$$

Tangensfunktion im rechtwinkligen magentafarbenen Teildreieck CDF

$$\tan 60^0 = \frac{3e}{\overline{CF}}$$

$$\tan 60^0 = \sqrt{3}$$

$$\sqrt{3} = \frac{3e}{\overline{CF}} \qquad \left| \cdot \overline{CF}\right.$$

$$\sqrt{3}\cdot\overline{CF} = 3e \qquad \left| : \sqrt{3}\right.$$

$$\overline{CF} = \frac{3e}{\sqrt{3}}$$

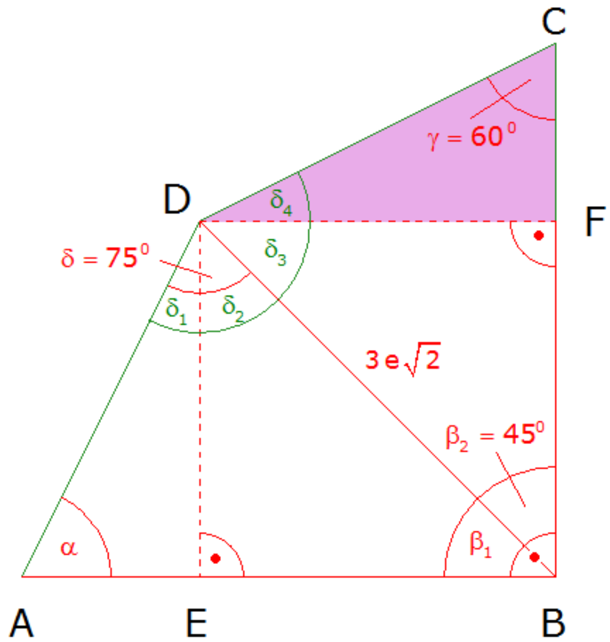
$$\overline{CF} = \frac{3e\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}}$$

Nenner rational machen

$$\overline{CF} = \frac{3e\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}}$$

$$\overline{CF} = \frac{3e\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}}$$

$$\overline{CF} = \frac{3e\cdot\sqrt{3}}{3}$$



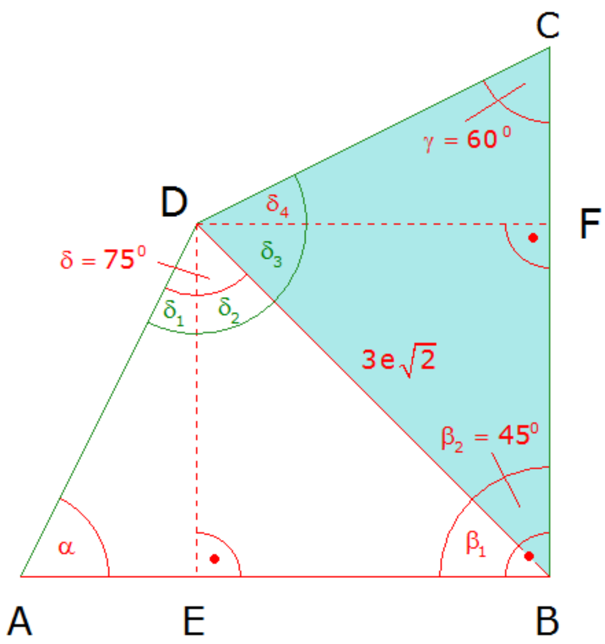
Lösung 2013 W1b:

$$\overline{CF} = \frac{3e \cdot \sqrt{3}}{3}$$

$$\overline{CF} = \frac{3 \cdot e \cdot \sqrt{3}}{3}$$ Bruch kürzen

$$\underline{\overline{CF} = e\sqrt{3}}$$

11. Berechnung der Strecke $\overline{BC}$:



$$\overline{BC} = \overline{CF} + \overline{BF}$$

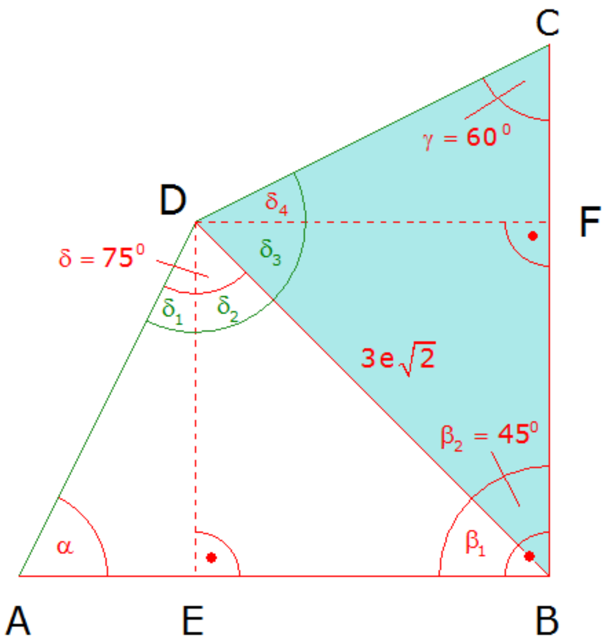
$$\overline{BC} = e\sqrt{3} + 3e$$

$$\overline{BC} = e\sqrt{3} + 3e$$ gemeinsamen Faktor ausklammern

$$\overline{BC} = e\left(\sqrt{3} + 3\right)$$

$$\underline{\overline{BC} = e\left(\sqrt{3} + 3\right)}$$

12. Berechnung der Dreiecksfläche A_{BCD}:



$$A_{BCD} = \frac{g \cdot h}{2} = \frac{\overline{BC} \cdot \overline{DF}}{2}$$ Flächenformel allgemeines Dreieck

$$A_{BCD} = \frac{e\left(\sqrt{3} + 3\right) \cdot 3e}{2}$$

$$A_{BCD} = \frac{3e^2\left(\sqrt{3} + 3\right)}{2}$$

$$\underline{A_{BCD} = 1{,}5\,e^2\left(\sqrt{3} + 3\right)}$$

Lösung 2013 W1b:

13. Berechnung der Vierecksfläche A_{ABCD}:

$$A_{ABCD} = A_{ABD} + A_{BCD}$$

$$A_{ABCD} = 1{,}5e^2\left(\sqrt{3}+3\right) + 1{,}5e^2\left(\sqrt{3}+3\right)$$

$$A_{ABCD} = 3e^2\left(\sqrt{3}+3\right)$$

$$A_{ABCD} = 3e^2\left(3+\sqrt{3}\right)$$